JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 15, No. 4, July–August 1992

Approach Guidance in a Downburst

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We present a nonlinear feedback law for safe aircraft penetration through downbursts on landing approach. It is based on following a nominal approach path relative to the ground, subject to a minimum airspeed constraint. The guidance design uses a point-mass aircraft model and considers the flight path to be in the vertical plane. We modify the slack variable transformation method to handle the airspeed constraint and apply the nonlinear dynamic inversion technique for the guidance law synthesis. Nonlinear numerical simulation results are presented for three different downburst histories. The main features of the proposed guidance logic are normal behavior when there is no wind, tracking of the nominal path in the presence of downbursts, and insensitivity to different downburst models. Piloting implications and other aspects of landing approach in downbursts are discussed.

Introduction

THE tragic crash of Delta Airlines Flight 191 attracted much attention to the danger of downbursts. This accident occurred on August 2, 1985, during an attempted landing at the Dallas/Ft. Worth Airport. Postaccident analysis by Bach and Wingrove[1] reconstructed the details of weather conditions and pilot responses and called for a re-examination of the conventional piloting strategy of maintaining airspeed in a downburst encounter on approach landing.

A downburst is a mass of cold air that descends to the ground in a column.[2] It only lasts about 5 to 10 min, but during that time, it creates the most threatening types of windshears and downdrafts.[3] Downbursts are the suspected cause of several aircraft accidents and continue to pose serious threats to safe aviation.

Studies have been carried out on different aspects of downbursts. Avoiding a downburst is the best strategy, yet a downburst is difficult to detect until the aircraft starts into it. Observation projects have provided much insight into the structure and characteristics of downbursts. Based on this information, various downburst models have been developed and used in simulations and system design. Reliable downburst detection and warning, especially by airborne "forward-look" systems, is still the subject of research.[4] Probably the most effective approach to accident prevention is still pilot awareness and training. The study of control laws is therefore important since it suggests piloting strategies for crew training and assists in the development of autopilots.

Important efforts have been made in the past to study landing approach in downbursts. Miele et al.[5] studied it by using a dynamic optimization method. The main term of their cost functional penalized the deviations from the nominal altitude path. Other terms included a penalty on airspeed deviation. They also designed guidance laws to approximate the optimal results. Psiaki and Stengel[6] conducted a comprehensive optimization study and determined the limiting windshear cases for safe landing penetration. The integral part of the cost functional penalized the flight-path deviation perpendicular to the glide slope whereas the deviations along the glide slope, including the deviation from inertial speed, were minimized in the terminal term. Both Bray[7] and Hinton[8] used simulation techniques and confirmed the severe consequences of a delayed response when a downburst is encountered. Hinton also reported the advantages of using a "forward-look" windshear sensor. Psiaki and Park[9] used a simplified first-order aircraft model and proposed a thrust law that maintains the minimum of airspeed and inertial speed at the nominal value.

Despite these investigations, there is still a need for an approach landing strategy with the following features. 1) Due to the difficulties in detecting a downburst, a good strategy that can alert the pilot should behave as usual when there is no wind, and respond correctly in the presence of a downburst. 2) Even though many downburst models have been proposed, none of them can describe the actual situation exactly, and therefore a good strategy should perform well in different wind profiles. 3) A feasible strategy should only require feedback quantities that can be measured onboard.

This paper examines a guidance concept that maintains the nominal approach path with respect to the inertial ground with a minimum airspeed constraint, a guidance logic that promises to achieve the aforementioned desired features. A point-mass aircraft model is used, and flight in the vertical plane is considered. A nonlinear dynamic inversion technique is combined with a slack variable transformation to design feedback guidance laws, and nonlinear simulation is then used for verification. The emphasis of this paper is on the feasibility of the proposed concept, thus not all of the details of practical autopilot design are considered.

The remainder of the paper is divided into five sections that outline the equations of motion, formulate the approach guidance problem, explain the control system design, discuss the simulation results, and, finally, present our conclusions.

Equations of Motion

A point-mass aircraft model is adequate for the current study since the time scale of the flight control problem in question is on the order of 1 min. In comparison, the time required to reach a certain value of angle of attack is just a few seconds. Therefore, we can use the angle of attack as a control and describe the aircraft as a point mass. The nonlinear point-mass model represents a phugoid approximation.

Flight in the vertical plane is considered. The actual flight is three dimensional, and an aircraft can detour laterally. If a downburst is detected far ahead, the aircraft can just avoid it.

Received Oct. 9, 1990; revision received Aug. 26, 1991; accepted for publication Aug. 28, 1991.

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Most of the time, however, a downburst can be sensed only when the aircraft is already close to it, and the benefits of lateral detouring are not at all clear. In fact, the pilot should concentrate on the vertical flight. In any case, the vertical motion of the aircraft is virtually the same as in the two-dimensional flight (see Chap. 8 of Ref. 10).

The equations of motion can be derived by using D'Alembert's principle. As shown by Miele et al.,[5] wind components show up explicitly in a frame of reference moving with the air mass. In this reference frame, the effects of wind rates can be described as D'Alembert forces. As a result, the equations of motion are as follows:

$$x' = V\cos\gamma + W_x = V_e \cos\gamma_e \tag{1}$$

$$h' = V\sin\gamma + W_h = V_e \sin\gamma_e \tag{2}$$

$$mV' = T\cos(\alpha+\delta) - D - mg\sin\gamma - mW_x'\cos\gamma - mW_h'\sin\gamma \tag{3}$$

$$mV\gamma' = T\sin(\alpha+\delta) + L - mg\cos\gamma - mW_h'\cos\gamma + mW_x'\sin\gamma \tag{4}$$

where the prime ( )′ represents a differentiation with respect to time, x and h are the horizontal and vertical coordinates of the aircraft, V is the airspeed, γ is the flight-path angle relative to the air mass, V_e is the inertial speed, γ_e is the absolute flight-path angle, α is the angle of attack, T is the thrust, δ is the thrust inclination angle, L and D are the lift and drag forces, m is the mass of the aircraft, and W_x and W_h are the horizontal and vertical wind components, respectively. In this definition, $W_x > 0$ is tail wind, and $W_h > 0$ is updraft. There are two controls: α and T. The functional relations of the aerodynamic forces T, L, and D formulated by Miele et al.[5] for the B-727 aircraft are used here.

To facilitate the guidance law design, we introduce some physical combinations of the previous variables. Let us start with two windshear factors:

$$f \triangleq \frac{dW_x}{dx}\cos\gamma + \frac{dW_h}{dx}\sin\gamma \tag{5}$$

$$p \triangleq \frac{dW_h}{dx}\cos\gamma - \frac{dW_x}{dx}\sin\gamma \tag{6}$$

where, if the wind field is assumed to be stationary,

$$\frac{dW_x}{dx} = \frac{\partial W_x}{\partial x} + \frac{\partial W_x}{\partial h}\frac{dh}{dx}$$

$$\frac{dW_h}{dx} = \frac{\partial W_h}{\partial x} + \frac{\partial W_h}{\partial h}\frac{dh}{dx}$$

We then introduce two normalized forces:

$$U \triangleq \frac{T\cos(\alpha+\delta) - D - mg\sin\gamma}{m(V\cos\gamma + W_x)} \tag{7}$$

$$F \triangleq \frac{T\sin(\alpha+\delta) + L - mg\cos\gamma}{m(V\cos\gamma + W_x)} \tag{8}$$

Using the horizontal distance x as the independent variable, the equations of motion become

$$\frac{dh}{dx} = \tan\gamma_e = \frac{V\sin\gamma + W_h}{V\cos\gamma + W_x} \tag{9}$$

$$\frac{dV}{dx} = U - f \tag{10}$$

$$V\frac{d\gamma}{dx} = F - p \tag{11}$$

$$\frac{dt}{dx} = \frac{1}{V\cos\gamma + W_x} \tag{12}$$

and it is easy to prove the following:

$$\frac{dV_e}{dx} = U\cos(\gamma_e - \gamma) + F\sin(\gamma_e - \gamma) \tag{13}$$

$$V_e\cos^2\gamma_e\frac{d^2h}{dx^2} = -U\sin(\gamma_e - \gamma) + F\cos(\gamma_e - \gamma) \tag{14}$$

Problem Formulation

The nominal approach path is defined with altitude and inertial speed as functions of horizontal distance. A safe approach requires that the touchdown position be close to the specified location and that the inertial speed and sink rate at touchdown be reasonable. A control logic following this nominal path has several desirable features (Ref. 11 discusses the advantages of following a nominal altitude trajectory defined as a function of horizontal distance x). In particular, this logic exhibits normal behavior when there is no wind and responds in a safe manner to the presence of a downburst. On the other hand, airspeed must be above a certain minimum level to insure controllability and to avoid stall.

Aircraft can usually follow the nominal landing path, as demonstrated every day by thousands of aircraft. With a point-mass aircraft model in the vertical plane, thrust and angle of attack are the two controls. The two outputs are altitude and inertial speed. However, the minimum airspeed constraint and control saturations affect the aircraft performance. The minimum airspeed constraint may prevent the maintenance of nominal inertial speed, as in a strong tail wind. Thrust has both an upper and a lower bound, and these bounds determine the amount of downburst energy that an aircraft can handle. A jet engine has a time constant of several seconds, so that a desired energy change rate may not be achievable. In addition, stall limits proper energy distribution. As a result of these limitations, some downbursts may not be survivable.

Having established the possibilities and limitations, we now formulate the problem mathematically. The approach guidance problem is to regulate the following two outputs to zero:

$$y_1(x) = h(x) - h_n(x) \tag{15}$$

$$y_2(x) = V_e(x) - V_{en}(x) \tag{16}$$

subject to various control bounds and a minimum airspeed constraint:

$$V \geq V_{\min} \tag{17}$$

where $h_n(x)$ and $V_{en}(x)$ describe the nominal landing path discussed next.

The nominal approach landing trajectory has two parts. In the glide approach part, the aircraft follows a constant inertial flight-path angle while flying at a constant inertial speed. In the flare part, the inertial speed is constant, but the inertial flight-path angle varies as a linear function of horizontal distance.[5]

If the switch from glide to flare is assumed to occur at $x = x_{sw}$ and $h = h_{sw}$, the nominal approach path is described by

$$h_n(x) = \begin{cases} h_n(x_0) + \tan\gamma_{e0}(x - x_0), & 0 \leq x < x_{sw} \\ h_{sw} + \tan\gamma_{e0}(x - x_{sw}) + D(x - x_{sw})^2, & x_{sw} \leq x \leq t_{td} \end{cases} \tag{18}$$

where

$$D = \frac{\tan^2 \gamma_{e0} - \tan^2 \gamma_{ef}}{4h_{sw}} \tag{19}$$

Inertial speed is constant throughout the whole approach landing:

$$V_{en} = V_{e0} \tag{20}$$

The flare starting position x_{sw}, touchdown distance x_{td}, and the time of flight t_f for the nominal approach landing are

$$x_{sw} = \frac{h_{sw} - h_0}{\tan \gamma_{e0}} \tag{21}$$

$$x_{td} = x_{sw} - \frac{2h_{sw}}{\tan \gamma_{ef} + \tan \gamma_{e0}} \tag{22}$$

$$t_f = \frac{x_{sw}}{V_{e0} \cos \gamma_{e0}} + \frac{1}{2DV_{e0}} \int_{\tan \gamma_{e0}}^{\tan \gamma_{ef}} \sqrt{1+s^2}\, ds \tag{23}$$

where V_{e0} is the nominal inertial speed, γ_{e0} is the nominal inertial flight-path angle in the glide approach part, and γ_{ef} is the nominal inertial flight-path angle at touchdown.

Control System Design

Without the airspeed constraint, there are exactly two controls for two outputs, and the nonlinear dynamic inversion technique applies.[12] For example, the linearized output dynamics may be specified as follows:

$$\frac{d^2 y_1}{dx^2} + K_{h1} \frac{dy_1}{dx} + K_{h2} y_1 = 0 \tag{24}$$

$$\frac{dy_2}{dx} + K_v y_2 = 0 \tag{25}$$

where the various K are some constants to be chosen; K_{h1} and K_v are in 1/ft, and K_{h2} is in 1/ft^2. As required by nonlinear inversion technique, enough differentiations are needed to reach controls U and F.

We can introduce some new quantities to facilitate the discussion. Define

$$Q_h \triangleq V_e \cos^2 \gamma_e \left[\frac{d^2 h_n}{dx^2} - K_{h1} \left(\tan \gamma_e - \frac{dh_n}{dx} \right) - K_{h2}(h - h_n) \right] \tag{26}$$

$$Q_v \triangleq -K_v (V_e - V_{e0}) \tag{27}$$

$$\Delta\gamma \triangleq \gamma_e - \gamma \tag{28}$$

and then from Eqs. (13–16), (24), and (25),

$$U \cos \Delta\gamma + F \sin \Delta\gamma = Q_v \tag{29}$$

$$-U \sin \Delta\gamma + F \cos \Delta\gamma = Q_h \tag{30}$$

which gives the following unconstrained feedback guidance laws:

$$U^* = Q_v \cos \Delta\gamma - Q_h \sin \Delta\gamma \tag{31}$$

$$F^* = Q_v \sin \Delta\gamma + Q_h \cos \Delta\gamma \tag{32}$$

The minimum airspeed constraint may preclude control efforts from achieving the desired dynamics of Eqs. (24) and (25), and thus the guidance laws U^* and F^* need to be altered to incorporate the airspeed constraint.

The slack variable transformation technique[13] in open-loop optimization is modified to handle the inequality constraint of Eq. (17) in feedback system design. In an open-loop case, this technique introduces an auxiliary state z and an auxiliary control w as follows:

$$V - V_{\min} = \tfrac{1}{2} z^2 \tag{33}$$

$$\frac{dz}{dx} = w$$

and V can be replaced by z in the optimization process. Since it takes one differentiation of Eq. (17) to reach control U, the airspeed inequality constraint is of first order, and only one pseudostate is needed. In essence, the slack variable transformation method converts an inequality constraint into an equality constraint. The equality constraint is satisfied through numerical iterations over global information. However, no iterations are possible, and only local information is available in a feedback system. Therefore, the slack variable method needs some modification. One approach is to satisfy Eq. (33) by regulating some quantity to zero. Specifically, assume Eq. (17) is satisfied initially and define

$$y_3 \triangleq V - V_{\min} - \tfrac{1}{2} z^2 \tag{34}$$

$$\frac{dz}{dx} \triangleq w \tag{35}$$

where

$$z_0 \triangleq z(x_0) = \sqrt{2\left[V(x_0) - V_{\min}\right]} \tag{36}$$

thus $y_3(x_0) = 0$.

Then, regulating y_3 to zero keeps airspeed above the minimum value. Again, this can be accomplished by specifying the desired output dynamics as follows:

$$\frac{dy_3}{dx} + K_y y_3 = 0 \tag{37}$$

where K_y is some constant to be chosen.

Expanding Eq. (37) gives

$$U = f - K_y (V - V_{\min}) + z(w + \tfrac{1}{2} K_y z) \tag{38}$$

The constrained guidance law is synthesized as follows. Equation (31) gives the desired U in a feedback form when there is no airspeed constraint. One can choose w in Eq. (38) so that U matches U^* as closely as possible. If z is large enough,

$$w = -\tfrac{1}{2} K_y z + 1/z \left[U^* + K_y (V - V_{\min}) - f\right] \tag{39}$$

The other control F can just be equal to F^* of Eq. (32).

There is, however, a possible singularity in the w solution. When z becomes increasingly small, the computed w from Eq. (39) approaches infinity. Since z has to be computed as part of the control law, and the implementation of Eq. (35) in a computer needs a smooth w, we require that w remain finite when z becomes very small.

To determine what z is small, we normalize z by its initial value:

$$\sigma \triangleq \frac{z}{z_0} \tag{40}$$

Thus $\sigma(x_0) = 1$, and Eq. (39) becomes

$$w = -\tfrac{1}{2} K_y z_0 \sigma + \frac{1}{\sigma} \frac{U^* + K_y (V - V_{\min}) - f}{z_0} \tag{41}$$

To make w bounded, one can approximate $1/\sigma$ by a function that is the same as $1/\sigma$ if σ is large but remains finite when σ approaches zero. For example, the following function is a choice:

$$W(\sigma) \triangleq \begin{cases} \dfrac{1}{\sigma} & \sigma > \Delta \\ \dfrac{1}{\Delta} & \sigma \le \Delta \end{cases} \tag{42}$$

where the dimensionless parameter Δ ($0 < \Delta \le 1$) will be chosen later, and thus

$$w = -\tfrac{1}{2} K_y z_0 \sigma + W(\sigma) \frac{U^* + K_y(V - V_{\min}) - f}{z_0} \tag{43}$$

which expresses the pseudocontrol w in a feedback form.

Combining Eqs. (38) and (43) determines U in a feedback form:

$$\begin{aligned} U &= U^* W_b(\sigma) + \left[f - K_y(V - V_{\min})\right]\left[1 - W_b(\sigma)\right] \\ &= U^* W_b(\sigma) + U_V\left[1 - W_b(\sigma)\right] \end{aligned} \tag{44}$$

where

$$W_b(\sigma) \triangleq \sigma W(\sigma) = \begin{cases} 1 & \sigma > \Delta \\ \dfrac{\sigma}{\Delta} & \sigma \le \Delta \end{cases} \tag{45}$$

$$U_V \triangleq f - K_y(V - V_{\min})$$

where U_V is precisely the feedback law to maintain $V = V_{\min}$, since Eq. (10) gives

$$\frac{\mathrm{d}(V - V_{\min})}{\mathrm{d}x} + K_y(V - V_{\min}) = U - U_V$$

and U^* is the feedback law to follow the inertial speed.

Therefore, the guidance law U contains a weighted sum of two terms. The first term directs the aircraft to follow the nominal inertial speed, whereas the second term maintains $V = V_{\min}$. The weighting functions shift more control effort toward the first term when σ is large or the airspeed is well above the minimum value and increase the second term if σ becomes small or if the airspeed is approaching the minimum value.

The σ can be calculated directly. Equations (35), (40), and (43) suggest

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x} = -\tfrac{1}{2} K_y \sigma + \frac{W(\sigma)}{2} \frac{U^* + K_y(V - V_{\min}) - f}{V_0 - V_{\min}} \tag{46}$$

and $\sigma(x_0) = 1$.

The variable σ is a dynamic indicator of how close the airspeed is to its minimum constraint. Since we try to regulate y_3 to zero in Eq. (34), z is related to $V - V_{\min}$. The σ is a normalized version of z and thus decreases when airspeed approaches the minimum value. In addition, σ increases when the aircraft is capable of following the nominal path while satisfying the minimum airspeed constraint. This is clear from Eq. (46) where, when the unfavorable wind intensity decreases (f becomes smaller), $\mathrm{d}\sigma/\mathrm{d}x$ increases. Since $V \ge V_{\min}$ is only a first-order constraint, σ should be close to but lead $V - V_{\min}$.

The possibility of σ becoming negative needs to be addressed. Choosing the positive square root in Eq. (36) guarantees the positiveness of σ only in theory if the aircraft has the capability to maintain the minimum airspeed. In practical numerical calculations, σ may possibly become negative. When this happens, the airspeed is around the minimum airspeed level $V_{\min}$, and the aircraft should concentrate on maintaining this minimum airspeed. We therefore impose the following condition in the control algorithm: Whenever σ becomes negative, set $\sigma = 0$. This condition also causes the guidance logic to follow the nominal path as soon as the wind situation becomes favorable.

Once U and F are determined, we can calculate the original controls T and α from Eqs. (7) and (8). For example, if

$$U_1 \triangleq mU(V \cos\gamma + W_x) + mg \sin\gamma \tag{47}$$

$$F_1 \triangleq mF(V \cos\gamma + W_x) + mg \cos\gamma \tag{48}$$

angle of attack α can be determined from

$$(U_1 + D)\sin(\alpha + \delta) - (F_1 - L)\cos(\alpha + \delta) = 0 \tag{49}$$

and the thrust T is given by

$$T = (U_1 + D)\cos(\alpha + \delta) + (F_1 - L)\sin(\alpha + \delta) \tag{50}$$

Feedback control laws must observe the physical limitations on T and α. If the angle of attack computed from Eq. (49) exceeds the maximum value, the maximum value is used to avoid stall: If $\alpha > \alpha_{\max}$, then $\alpha = \alpha_{\max}$. A typical bound on the rate of change of angle of attack is about 3 deg/s. This bound does not pose a strong limit in the current problem of one minute time interval and thus is omitted. It can be easily added if necessary.

Miele et al.[5] expressed the thrust as a product of maximum thrust and thrust setting level:

$$T = \beta T_{\max}$$

The thrust setting input β is subject to both magnitude bounds and rate bounds:

$$\beta_{\text{low}} \le \beta \le 1, \qquad -\beta'_{\max} \le \beta' \le \beta'_{\max}$$

and these bounds apply to adjust the desired β from Eq. (50).

The following values are assumed in the simulations:

$$\alpha_{\max} = 17.2 \text{ deg}, \quad \beta_{\text{low}} = 0.2, \quad \beta'_{\max} = 0.3 \text{ s}^{-1}$$

Various feedback gain K determine the closed-loop response characteristics. Although there are many ways to select these parameters, we use a simple order of magnitude analysis. Numerically, terms added to each other should be of comparable magnitude to retain accuracy. Physically, as in the case of Eq. (7), thrust overcomes the drag and a component of the weight, and thus T, D, and $mg \sin\gamma$ are roughly on the same order of magnitude. Any variations of Eqs. (7) and (8) are preferably on a similar order of magnitude. Therefore, if $[X]$ denotes the order of magnitude of variable X, Eqs. (7) and (8) suggest

$$[U] = [g \tan\gamma_0 / V_0] \quad \text{and} \quad [F] = [g/V_0]$$

and since γ_0 is small, Eqs. (26–30) and Eq. (38) give

$$[Q_h] = [F] = [K_{hl}][V_0\gamma_0]$$

$$[Q_v] = [U] = [K_v][V_0]$$

$$[U] = [K_y][V_0]$$

Therefore,

$$[K_y] = [K_v] = [g\gamma_0/V_0^2] = [3 \times 10^{-5}]$$

$$[K_{hl}] = [g/V_0^2\gamma_0] = [0.01]$$

where $\gamma_0 = -3$ deg and $V_0 = 239.7$ ft/s.

Once K_{h1} is selected, K_{h2} determines the closed-loop roots of the linearized system of Eq. (24). In the following simulations,

$$K_{h1} = 5\sqrt{2} \times 10^{-3}\ \text{ft}^{-1}, \qquad K_{h2} = 2.5 \times 10^{-5}\ \text{ft}^{-2}$$

$$K_v = 3 \times 10^{-5}\ \text{ft}^{-1}, \qquad K_y = 3 \times 10^{-4}\ \text{ft}^{-1}$$

where K_y is an order of magnitude larger than K_v to keep the minimum airspeed constraint.

A proper value of Δ represents a compromise between maintaining the minimum airspeed limit and following the nominal landing path. A large Δ causes the control to act as soon as the airspeed begins to deviate toward the minimum value. The airspeed constraint is then well maintained at the expense of a larger deviation from the nominal path. On the other hand, a small Δ concentrates the control effort on keeping the nominal path until the violation of the minimum airspeed constraint is imminent. Due to the engine time constant, this may cause undershoot of the minimum airspeed value. Thus, the selection of a suitable Δ value requires the assessment of practical downburst characteristics. In the simulations that follow, a value of $\Delta = 0.7$ is used; this diverts control efforts for airspeed when $V - V_{\min} \leq 0.5(V_0 - V_{\min})$.

The proposed guidance law can be implemented in practice. This law requires for feedback the measurements or estimates of airspeed V, flight-path angle relative to air mass γ, inertial speed and sink rate, horizontal distance and altitude, and windshear factor f. The measurements of wind components W_x and W_h are not necessary if inertial speeds are available. The Global Positioning Systems (GPS) provides accurate inertial speeds and positions at an increasingly low cost.[14] The windshear factor f is related to the horizontal part of the Bowles "F-factor," the measurement of which has been heavily researched.[15]

It is possible to obtain simplified guidance laws. For example, selecting $K_{h2} = 0$ will eliminate the need to measure altitude h. Since airspeed constraint is of first order, σ is closely related and thus can be approximated by $V - V_{\min}$; this will drop the requirement to measure f. Finally, if one chooses not to control the inertial speed along the glide path, one can use Eqs. (30) and (38) to design new guidance laws. Although details are omitted for clarity, simulations indicate that these simplifications work reasonably well.

Simulations and Discussions

Three wind histories of horizontal wind component alone, vertical component alone, and a general downburst profile are used in the simulation. The horizontal wind component (HW) is described by

$$W_x = \begin{cases} -50 \sin\left(\dfrac{2x\pi}{4000}\right) & \text{if } x \leq 8000 \text{ ft} \\ 0 & \text{else} \end{cases}$$

$$W_h = 0$$

and the vertical wind component (VW) is described by

$$W_x = 0$$

$$W_h = \begin{cases} 20\left[\cos\left(\dfrac{2x\pi}{8000}\right) - 1\right] & \text{if } x \leq 8000 \text{ ft} \\ 0 & \text{else} \end{cases}$$

where all of the wind components are in ft/s.

The general downburst profile (GW) is modeled by two rings of the simplified ring-vortex downburst model developed in Appendix A of Ref. 10. Appendix B of Ref. 16 contains the analytical expressions. Table 1 lists the parameters obtained in Ref. 10 by a least square fit into the DFW (Dallas/Ft. Worth) downburst data.[1]

The initial conditions used in the simulations are the following[5]:

$$x(0) = 0$$

$$h(0) = 600 \text{ ft}$$

$$V(0) = 239.7 \text{ ft/s}$$

$$\gamma(0) = -3 \text{ deg}$$

Table 1 DFW downburst parameters

Ring one	Ring two	Unit
$\Gamma_1 = 444167.3$	$\Gamma_2 = 78941.5$	ft²/s
$H_1 = 2258.8$	$H_2 = 2215.3$	ft
$X_1 = 9204.3$	$X_2 = 9681.9$	ft
$R_1 = 3574.5$	$R_2 = 1591.3$	ft
$R_{c1} = 400.0$	$R_{c2} = 200.0$	ft

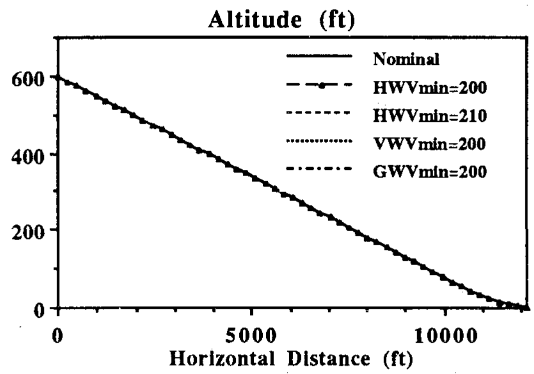

Fig. 1 Altitude histories.

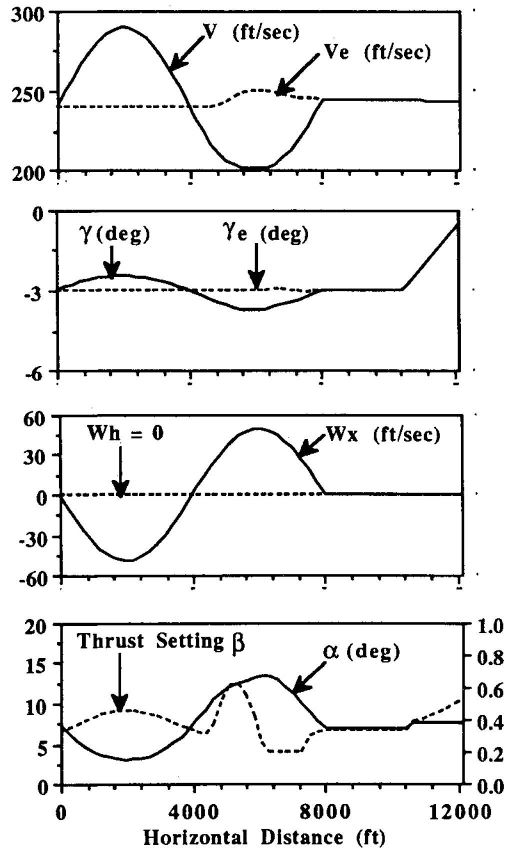

Fig. 2 In horizontal wind with $V_{\min} = 200$ ft/s.

$$h_{sw} = 50 \text{ ft}$$

The aircraft is assumed to be in an equilibirum state before it encounters any wind, which corresponds to

$$\alpha_0 = 7.37 \text{ deg}, \qquad \beta_0 = 0.33228$$

Some useful nominal values are

$$x_{td}^* = 12{,}130.36 \text{ ft}, \qquad t_f^* = 50.67 \text{ s}$$

The simulation uses a Newton-Raphson iteration scheme to solve Eq. (49) and Hamming's formula[17] with 200 divisions of the interval $[0, x_{td}]$ for integration. The simulation terminates when the altitude is zero.

Four simulation cases are presented. The guidance logic is tested in the horizontal wind (HW) with $V_{\min} = 200{,}210$ ft/s, in the vertical wind (VW) with $V_{\min} = 200$ ft/s, and in the general downburst wind profile (GW) with $V_{\min} = 200$ ft/s. Figure 1 compares the simulated altitude histories of these four cases with the nominal path, and Figs. 2, 3, 4, and 5 present the corresponding airspeed, inertial speed, flight-path angles, and controls for these four cases. Table 2 documents the simulation cases and flight-time deviations from the nominal value. In all of the simulations, the aircraft land exactly on the specified position so that $x_{td} - x_{td}^* = 0$, but the inertial speed and sink rate at touchdown are different from the nominal values.

The simulation results provide much insight into landing approach through a downburst. The aircraft is able to maintain the nominal landing path in all three wind scenarios, at the expense of deviations of airspeed and/or air-relative flight-path angle. The guidance law is indeed insensitive to different downburst models. Control efforts vary a great deal from one downburst history to another, and therefore it is necessary to use both thrust and angle of attack actively. The flight-time deviations are acceptable. If a higher precision is necessary for flight time, inertial speed, and sink rate at touchdown, one can choose a larger K_v.

Specifically, a horizontal wind component mainly causes airspeed fluctuation while producing small variations in the air-relative flight-path angle (Figs. 2 and 3). The airspeed fluctuation is roughly 180 deg out of phase with the horizontal wind component. To compensate for the lift change caused by airspeed variation, the angle of attack varies 180 deg out of phase with the airspeed. The increasing headwind may add to the aircraft air-relative energy, but it reduces the aircraft's inertial energy, causing an increase in thrust. The aircraft speeds up in head wind and slows down in tail wind to keep the nominal inertial speed. It also climbs up slightly in head wind and pitches down slightly in tail wind.

A vertical wind component does not affect airspeed very much, thus causing just a small motion in angle of attack (Fig. 4). It mainly causes a deviation of the air-relative flight-path angle. The aircraft has to climb up with respect to the downward wind to maintain the nominal inertial course. The thrust has to respond dramatically to compensate for the inertial energy loss caused by the downdraft.

The general wind profile approximates the DFW downburst scenario. The vortex-ring model does not describe the downdraft component adequately but does present the basic features of a downburst. The key to successfully following the nominal landing path is to maintain inertial speed and flight-path angle, not air-relative quantities (see Fig. 5).

Simulations show that the minimum airspeed constraint level and control bounds limit aircraft performance in downbursts. When using a $V_{\min} = 210$ ft/s instead of 200 ft/s, the aircraft has a higher inertial speed (see Figs. 2 and 3). A smaller thrust setting is then needed to reduce the excess inertial speed. The lower thrust bound, however, prevents a rapid reduction of inertial energy, thus causing the aircraft to land sooner (Table 2), with higher inertial speed and sink rate at touchdown than in the nominal flight. In general, higher $V_{\min}$ re-

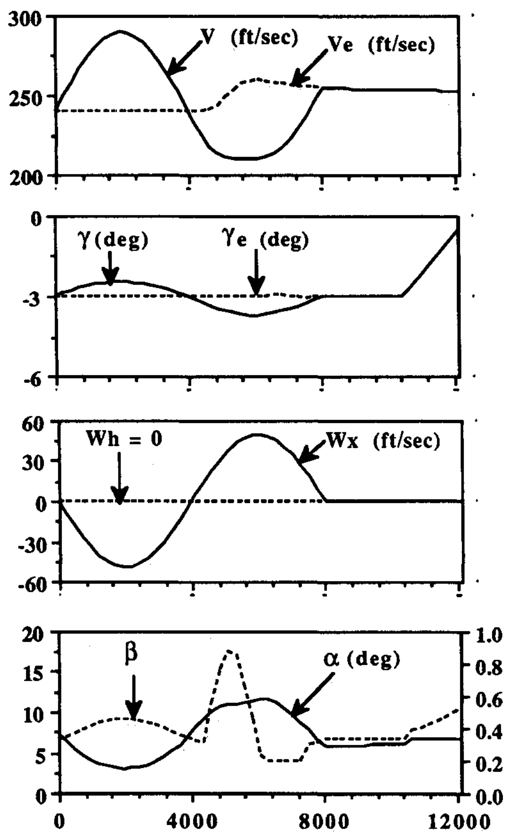

Fig. 3 In horizontal wind with $V_{\min} = 210$ ft/s.

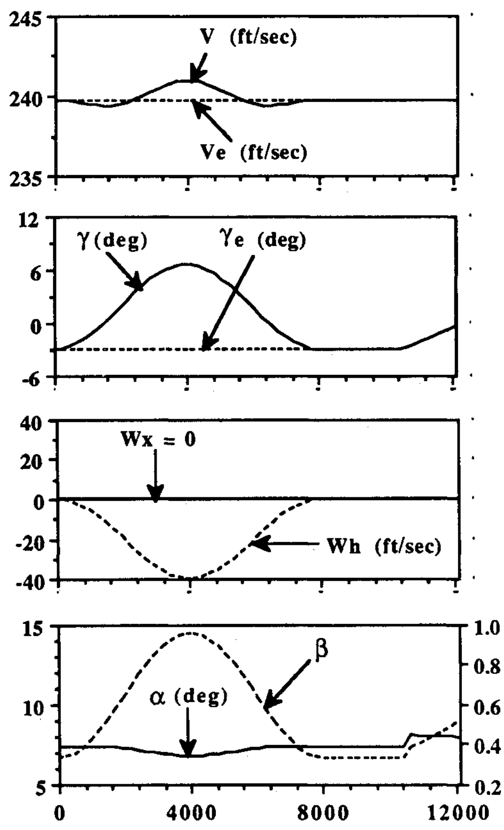

Fig. 4 In vertical wind with $V_{\min} = 200$ ft/s.

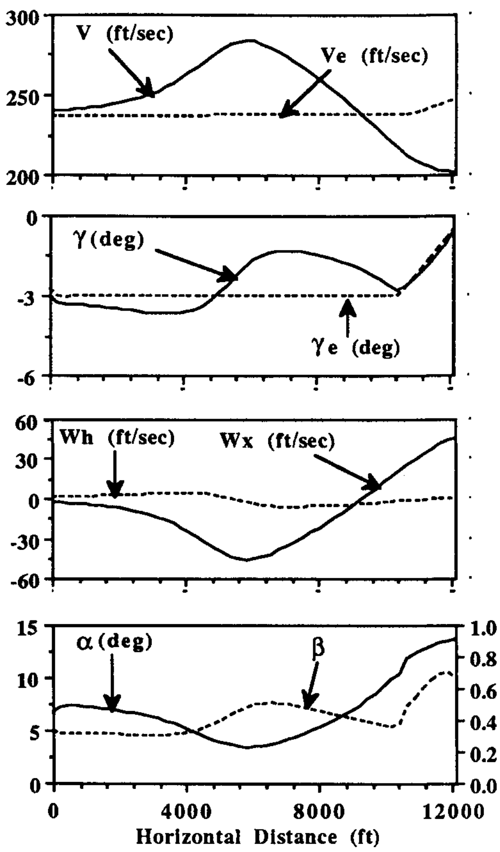

Fig. 5 In general wind with $V_{min} = 200$ ft/s.

quires more control effort for airspeed, thus leaving less for the nominal path following. With too low a V_{min}, however, one runs the risk of stalling the aircraft in a fast-changing horizontal wind. To establish a suitable V_{min} value, further studies will be needed on the downburst frequency spectrum and the aircraft engine time constant.

Simulations using stronger downdrafts and tail winds are also revealing. A large downdraft calls for a large thrust compensation and may cause a thrust saturation with the upper bound, resulting in lower inertial speed and sink rate at touchdown and a longer time of flight compared with the nominal path. A strong tailwind makes it difficult to maintain the minimum airspeed while maintaining inertial speed. As a result, inertial speed and sink rate at touchdown will be high, and time of flight will be shorter than the nominal value. Severe tail winds can saturate both the angle of attack and thrust, causing the aircraft to deviate from the nominal altitude path. Details of these simulations are omitted.

The simulation results establish the feasibility of the proposed landing guidance strategy. We now discuss related aspects, such as the differences between takeoff and landing flight in downbursts, the decision to proceed or to abort once an aircraft encounters a downburst on landing approach, further studies needed to complete the control law design, and the guidance system design method presented in this paper.

Safety concerns are the reason for the differences between takeoff flight and approach landing flight strategies. Since the aircraft is close to the ground, altitude is important for both takeoff and landing. Once airborne in takeoff flight, an aircraft tries to remain airborne to avoid ground impact.[16] On the other hand, an approach landing aircraft is going to interact with the ground. Therefore, a proper reference frame for takeoff is air relative, whereas a proper reference frame for approach landing must be inertial. As a result, air-relative energy or pseudoenergy[16] dominates the takeoff problem, whereas inertial energy and its distribution governs the approach landing flight. In addition, the same wind components may have

Table 2 Simulation results

Case	Description	V_{min} (ft/s)	$t_f - t_f^*$ (s)
1	Horizontal wind	200	−0.70
2	Horizontal wind	210	−1.77
3	Vertical wind	200	0.00
4	General wind	300	0.40

different effects on different flights. Tail wind and downdraft are the main dangers to an aircraft on takeoff, and increasing head wind and decreasing tail wind are considered to be "performance increasing." In approach landing flights, however, any wind component is undesirable. As a result of these differences, takeoff and landing flights have different needs for sensing and control system implementation. For example, although inertial speed is not necessary for implementing takeoff control laws, it is essential for implementing approach landing controllers. Because of the need to stay airborne, abort landing flight is similar to takeoff flight and is thus different from continued approach landing flight.

Since it performs normally when there is no wind, the proposed guidance logic system can be kept on during the entire approach landing flight. If the pilot believes that he has entered a downburst on approach, he has two choices: to abort the landing or to proceed. If the encounter altitude is high, it is prudent to abort. Abort landing requires high thrust for climbing out and will possibly fly the aircraft across the whole downburst. Due to the large engine time constant and low nominal thrust setting, it may not be possible to abort the landing if the encounter altitude is low. In this case, proceeding with the approach is safer. If the proposed logic is used during the entire approach, deviations in airspeed and air-relative flight-path angle can serve as indicators of the downburst intensity and thus provide a basis for detection and for making the decision to abort or to proceed. Should proceeding with the flight be preferred, the guidance logic has already performed correctly.

Further studies on the proposed guidance laws will be required for practical applications. These studies should use a higher-order aircraft model, incorporate random wind components, and consider three-dimensional flight. Due to the uncertainties in aircraft modeling, robustness analysis will be necessary.

The modified slack variable transformation technique is useful in general feedback control design with state inequality constraints for both linear and nonlinear systems. The selection and properties of the $W(\sigma)$ function will need further investigation for other applications. The order of magnitude analysis is useful in determining feedback coefficients in conjunction with nonlinear dynamic inversion techniques.

Conclusions

This paper confirms the feasibility of an aircraft approach guidance logic, which maintains the nominal altitude and inertial speed as functions of horizontal distance, under a minimum airspeed constraint. The design assumes a point-mass aircraft flying in the vertical plane, applies a transformation technique together with the nonlinear dynamic inversion to obtain feedback guidance laws, and uses nonlinear simulations to verify design objectives. The proposed guidance strategy exhibits normal behavior when there is no wind, maintains the nominal approach landing path in the presence of different downburst profiles, and is implementable for practical applications. Simulation results indicate that while the vertical wind component mainly perturbs the air-relative flight angle, the horizontal wind component affects both airspeed and air-relative flight-path angle, and, in short, a downburst causes fluctuations in aircraft inertial energy and its distribution. In general, one must use both thrust and angle of attack to compensate for the effects of a downburst. The choice of the minimum airspeed constraint presents a compromise between following the nominal path and preventing the aircraft from accidental stall. Control saturations determine the limits of the balancing effects of any control strategies, and some downbursts are not penetrable. We recommend that this guidance strategy be studied further by using a more complete aircraft model, random wind profiles, and pilot-in-the-loop simulation.

Acknowledgments

The authors thank H. Stalford for his encouragement, without which the paper would have taken another year to complete. We also thank A. Miele for kindly providing copies of his publications.

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